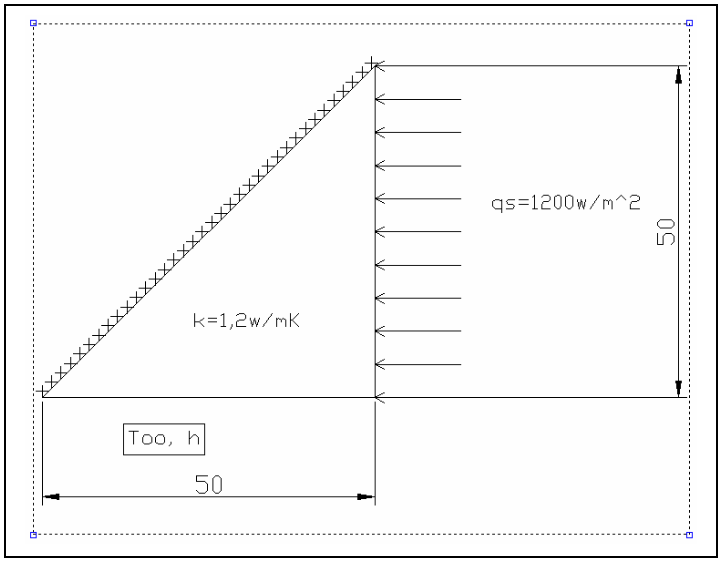
TRANSFERENCIA DE CALOR

FLUJO DE CALOR BIDIMENSIONAL

Planteamiento de los ejercicios.

Ejercicio 1 →



$h_i = 1000$ w/m^2°C.

$T_i = 450$ °C.

$C_{p1} = 700$ J/kgK.

$h = 25$ w/m^2°C.

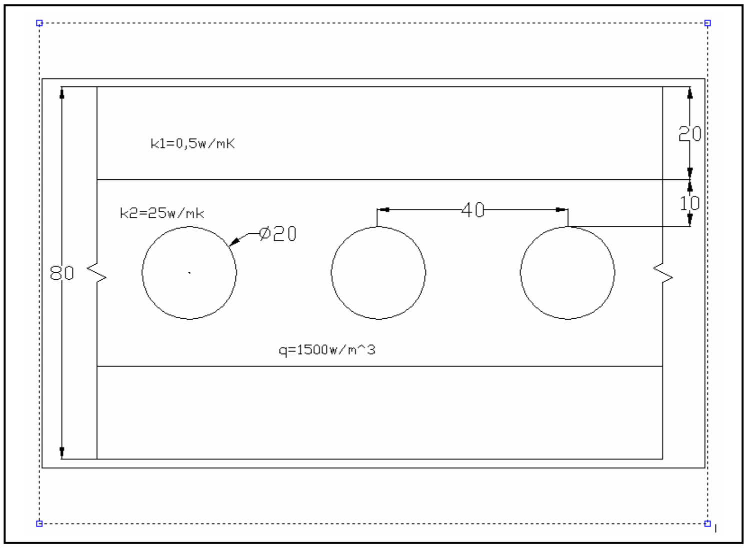
$T_{oo} = 27$°C.

$\rho_1 = 3000$ kg/m^3.

$\rho_2 = 3000$ kg/m^3.

$C_{p1} = 500$ J/kgK.

Ejercicio 2 →



Se pide:

a. Solución analítica y sus isotermas.

b. Solución por diferencia finita y sus isotermas.

c. Calor disipado por las caras convectivas.

d. Comparar solución analítica y diferencia finita.

e. Presentar isotermas para 5, 10, 15, 20, 25 y 30 minutos.

Ejercicio 1. METODO ANALITICO

Solución 1.

$$\theta = \left[Asen(\lambda y) + B\cos(\lambda y) \right]\left[Csenh(\lambda x) + D\cosh(\lambda x) \right]$$

Para $\frac{d\theta}{dx}_{x=0} + \frac{h\theta}{k}_{x=0} = 0$ tenemos lo siguiente:

$$0 = \left[Asen(\lambda_y) + B\cos(\lambda_y) \right]\lambda\left[C\cosh(\lambda 0) + Dsenh(\lambda 0) \right]$$

$$+\frac{h}{k}\left[Asen(\lambda y) + B\cos(\lambda y) \right]\left[Csenh(\lambda 0) + D\cosh(\lambda 0) \right]$$

$$0 = \left[Asen(\lambda y) + B\cos(\lambda y) \right]\lambda C + \frac{h}{k}\left[Asen(\lambda y) + B\cos(\lambda y) \right]D$$

$$0 = \lambda C + \frac{h}{k}D \qquad \rightarrow \qquad C = -\frac{h}{k\lambda}D$$

Para $\frac{d\theta}{dy}_{y=0} + \frac{h\theta}{k}_{y=0} = 0$ tenemos lo siguiente:

$$0 = \lambda\left[A\cos(\lambda 0) - Bsen(\lambda 0) \right]D\left[-\frac{h}{k\lambda}senh(\lambda_x) + \cosh(\lambda x) \right]$$

$$+\frac{h}{k}\left[Asen(\lambda 0) + B\cos(\lambda 0) \right]D\left[-\frac{h}{k\lambda}senh(\lambda x) + \cosh(\lambda x) \right]$$

$$0 = \lambda A + \frac{h}{k}B \qquad \rightarrow \qquad A = -\frac{hB}{k\lambda}$$

$$\theta = B\left[-\frac{h}{k\lambda}sen(\lambda_y) + \cos(\lambda_y) \right]D\left[-\frac{h}{k\lambda}senh(\lambda_x) + \cosh(\lambda x) \right]$$

$$= A\left[-\frac{h}{k\lambda}sen(\lambda y) + \cos(\lambda y) \right]\left[-\frac{h}{k\lambda}senh(\lambda x) + \cosh(\lambda x) \right]$$

Para $\frac{d\theta}{dy}_{y=L} = 0$ tenemos lo siguiente:

$$0 = A\lambda\left[-\frac{h}{k\lambda}\cos(\lambda L) - sen(\lambda L) \right]\left[-\frac{h}{k\lambda}senh(\lambda x) + \cosh(\lambda x) \right]$$

$$\frac{h}{k\lambda}\cos(\lambda L)+sen(\lambda L)=0 \quad \rightarrow \quad \frac{h}{k\lambda}\cos(\lambda L)=-sen(\lambda L) \quad \rightarrow \quad \frac{h}{k\lambda}=-\tan(\lambda L)$$

$\tan(\lambda L)+\dfrac{h}{k\lambda}=0$, $\quad u=\lambda L \quad \rightarrow \quad \tan(u)+\dfrac{Bi}{u}=0$, se obtienen 7 valores de la gráfica.

Para $\left.\dfrac{d\theta}{dx}\right|_{x=L}=-\dfrac{q}{k}$ tenemos lo siguiente:

$$-\frac{q}{k}=A\lambda\left[-\frac{h}{k\lambda}sen(\lambda y)+\cos(\lambda y)\right]\left[-\frac{h}{k\lambda}\cosh(\lambda L)+senh(\lambda L)\right]$$

$$C_n=A\lambda\left[-\frac{h}{k\lambda}\cosh(\lambda L)+senh(\lambda L)\right] \quad \rightarrow \quad -\frac{q}{k}=\sum_{n=1}^{\infty}C_n\left[-\frac{h}{k\lambda}sen(\lambda_n y)+\cos(\lambda_n y)\right]$$

$$-\frac{q}{k}\cos(\lambda_m y)=\sum_{n=1}^{\infty}C_n\left[-\frac{h}{k\lambda}sen(\lambda_n y)+\cos(\lambda_n y)\right]\cos(\lambda_m y),$$ integrando…

$$\int_{-L}^{0}-\frac{q}{k}\cos(\lambda_m y)dy=\int_{-L}^{0}\sum_{n=1}^{\infty}C_n\left[-\frac{h}{k\lambda}sen(\lambda_n y)+\cos(\lambda_n y)\right]\cos(\lambda_m y)dy$$

$$\int_{-L}^{0}-\frac{q}{k}\cos(\lambda_m y)dy=\int_{0}^{L}-\frac{q}{k}\cos(\lambda_m y)dy,$$ debido a que el coseno es una función par tenemos que:

$$\int_{-L}^{0}-\frac{q}{k}\cos(\lambda_m y)dy+\int_{0}^{L}-\frac{q}{k}\cos(\lambda_m y)dy=2\int_{0}^{L}-\frac{q}{k}\cos(\lambda_m y)dy$$

$$2\int_{0}^{L}-\frac{q}{k}\cos(\lambda_m y)dy=\int_{-L}^{0}\sum_{n=1}^{\infty}C_n\left[-\frac{h}{k}sen(\lambda_n y)+\cos(\lambda_n y)\right]\cos(\lambda_m y)dy+\int_{0}^{L}\sum_{n=1}^{\infty}C_n\left[-\frac{h}{k\lambda}sen(\lambda_n y)+\cos(\lambda_n y)\right]\cos(\lambda_m y)dy$$

$$-2\frac{q}{k}\int_{0}^{L}\cos(\lambda_m y)dy=\int_{-L}^{L}\sum_{n=1}^{\infty}C_n\left[-\frac{h}{k\lambda}sen(\lambda_n y)\cos(\lambda_m y)\right]+2\sum_{n=1}^{\infty}\int_{0}^{L}C_n\cos(\lambda_n y)\cos(\lambda_m y)dy$$

$$-2\frac{q}{k}\int_{0}^{L}\cos(\lambda_m y)dy=-\frac{h}{k\lambda}\left[\int_{-L}^{L}C_1 sen(\lambda_1 y)\cos(\lambda_1 y)dy+\ldots+\int_{-L}^{L}C_m sen(\lambda_m y)\cos(\lambda_m y)dy+\ldots+\int_{-L}^{L}C_n sen(\lambda_n y)\cos(\lambda_n y)\right]$$

$$+2\left[\int_{0}^{L}C_1\cos(\lambda_1 y)\cos(\lambda_m y)dy+\ldots+\int_{0}^{L}C_m\cos(\lambda_m y)\cos(\lambda_m y)dy+\ldots+\int_{0}^{L}C_m\cos(\lambda_n y)\cos(\lambda_m y)dy\right]$$

$$-2\frac{q}{k}\left[\frac{sen\lambda_n y}{\lambda_n}\right]_0^L=2\left[\frac{y}{2}+\frac{sen(2\lambda y)}{4\lambda}\right]_0^L=2\left[\frac{L}{2}+\frac{sen(2\lambda L)}{4\lambda}\right]C_n$$

$$-2\frac{q}{k\lambda_n}sen(\lambda_n L)=2\left[\frac{L}{2}+\frac{sen(2\lambda L)}{4\lambda}\right]C_n$$

$$C_n = \frac{-\,{}^{q}\!/_{k\lambda_n}\, sen(\lambda_n L)}{{}^{L}\!/_{2} + {}^{sen(2\lambda L)}\!/_{4\lambda}} = \frac{-4q\,sen(\lambda_n L)}{[2L\lambda + sen(2\lambda L)]k}$$

$$A_n = \frac{C_n}{\lambda_n\left[-\,{}^{h}\!/_{k\lambda_n}\cosh(\lambda_n L) + senh(\lambda_n L)\right]}$$

$$\theta_1 = \sum_{n=1}^{7} A_n\left[-\frac{h}{k\lambda}sen(\lambda y) + \cos(\lambda y)\right]\left[-\frac{h}{k\lambda}senh(\lambda x) + \cosh(\lambda x)\right]$$

Solución 2.

$$\theta = \left[Asen(\lambda x) + B\cos(\lambda x)\right]\left[Csenh(\lambda y) + D\cosh(\lambda y)\right]$$

Para $\left.\frac{d\theta}{dy}\right|_{y=0} + \left.\frac{h\theta}{k}\right|_{y=0} = 0$ tenemos lo siguiente:

$$0 = \left[Asen(\lambda x) + B\cos(\lambda x)\right]\lambda\left[C\cosh(\lambda 0) + Dsenh(\lambda 0)\right]$$
$$+\frac{h}{k}\left[Asen(\lambda x) + B\cos(\lambda x)\right]\left[Csenh(\lambda 0) + D\cosh(\lambda 0)\right]$$

$$0 = [Asen(\lambda_x) + B\cos(\lambda_x)]\lambda C + \frac{h}{k}[Asen(\lambda_x) + B\cos(\lambda_x)]D$$

$$0 = \lambda C + \frac{h}{k}D \qquad \rightarrow \qquad C = -\frac{h}{k\lambda}D$$

Para $\left.\frac{d\theta}{dx}\right|_{x=0} + \left.\frac{h\theta}{k}\right|_{x=0} = 0$ tenemos lo siguiente:

$$0 = \lambda\left[A\cos(\lambda 0) - Bsen(\lambda 0)\right]D\left[-\frac{h}{k\lambda}senh(\lambda_y) + \cosh(\lambda_y)\right]$$
$$+\frac{h}{k}\left[Asen(\lambda 0) + B\cos(\lambda 0)\right]D\left[-\frac{h}{k\lambda}senh(\lambda_y) + \cosh(\lambda_y)\right]$$

$$0 = \lambda A + \frac{h}{k}B \qquad \rightarrow \qquad A = -\frac{hB}{k\lambda}$$

$$\theta = B\left[-\frac{h}{k\lambda}sen(\lambda x) + \cos(\lambda x)\right]D\left[-\frac{h}{k\lambda}senh(\lambda y) + \cosh(\lambda y)\right] =$$
$$A\left[-\frac{h}{k\lambda}sen(\lambda x) + \cos(\lambda x)\right]\left[-\frac{h}{k\lambda}senh(\lambda y) + \cosh(\lambda y)\right]$$

Para $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$ tenemos lo siguiente:

$$0 = A\lambda\left[-\frac{h}{k\lambda}\cos(\lambda L) - sen(\lambda L)\right]\left[-\frac{h}{k\lambda}senh(\lambda y) + \cosh(\lambda y)\right]$$

$$\frac{h}{k\lambda}\cos(\lambda L) + sen(\lambda L) = 0 \quad \rightarrow \quad \frac{h}{k\lambda}\cos(\lambda L) = -sen(\lambda L) \quad \rightarrow \quad \frac{h}{k\lambda} = -\tan(\lambda L)$$

$\tan(\lambda L) + \frac{h}{k\lambda} = 0$, $\quad u = \lambda L \quad \rightarrow \quad \tan(u) + \frac{Bi}{u} = 0$, se obtienen 7 valores de la gráfica.

Para $\left.\frac{d\theta}{dy}\right|_{y=L} = -\frac{q}{k}$ tenemos lo siguiente:

$$-\frac{q}{k} = A\lambda\left[-\frac{h}{k\lambda}sen(\lambda x) + \cos(\lambda x)\right]\left[-\frac{h}{k\lambda}\cosh(\lambda L) + senh(\lambda L)\right]$$

$$C_n = A\lambda\left[-\frac{h}{k\lambda}\cosh(\lambda L) + senh(\lambda L)\right] \quad \rightarrow \quad -\frac{q}{k} = \sum_{n=1}^{\infty} C_n\left[-\frac{h}{k\lambda}sen(\lambda_n x) + \cos(\lambda_n x)\right]$$

$-\frac{q}{k}\cos(\lambda_m x) = \sum_{n=1}^{\infty} C_n\left[-\frac{h}{k\lambda}sen(\lambda_n x) + \cos(\lambda_n x)\right]\cos(\lambda_m x)$, integrando…

$$\int_{-L}^{0} -\frac{q}{k}\cos(\lambda_m x)dx = \int_{-L}^{0}\sum_{n=1}^{\infty} C_n\left[-\frac{h}{k\lambda}sen(\lambda_n x) + \cos(\lambda_n x)\right]\cos(\lambda_m x)dx$$

$\int_{-L}^{0} -\frac{q}{k}\cos(\lambda_m x)dx = \int_{0}^{L} -\frac{q}{k}\cos(\lambda_m x)dx$, debido a que el coseno es una función par tenemos que:

$$\int_{-L}^{0} -\frac{q}{k}\cos(\lambda_m x)dx + \int_{0}^{L} -\frac{q}{k}\cos(\lambda_m x)dx = 2\int_{0}^{L} -\frac{q}{k}\cos(\lambda_m x)dx$$

$$2\int_{0}^{L} -\frac{q}{k}\cos(\lambda_m x)dx = \int_{-L}^{0}\sum_{n=1}^{\infty} C_n\left[-\frac{h}{k}sen(\lambda_n x) + \cos(\lambda_n x)\right]\cos(\lambda_m x)dx + \int_{0}^{L}\sum_{n=1}^{\infty} C_n\left[-\frac{h}{k\lambda}sen(\lambda_n x) + \cos(\lambda_n x)\right]\cos(\lambda_m x)dx$$

$$-2\frac{q}{k}\int_{0}^{L}\cos(\lambda_m x)dx = \int_{-L}^{L}\sum_{n=1}^{\infty} C_n\left[-\frac{h}{k\lambda}sen(\lambda_n x)\cos(\lambda_m x)\right] + 2\sum_{n=1}^{\infty}\int_{0}^{L} C_n\cos(\lambda_n x)\cos(\lambda_m x)dx$$

$$-2\frac{q}{k}\int_{0}^{L}\cos(\lambda_m x)dx = -\frac{h}{k\lambda}\left[\int_{-L}^{L} C_1 sen(\lambda_1 x)\cos(\lambda_1 x)dx + \ldots + \int_{-L}^{L} C_m sen(\lambda_m x)\cos(\lambda_m x)dx + \ldots + \int_{-L}^{L} C_n sen(\lambda_n x)\cos(\lambda_n x)\right]$$

$$+2\left[\int_{0}^{L} C_1\cos(\lambda_1 x)\cos(\lambda_m x)dx + \ldots + \int_{0}^{L} C_m\cos(\lambda_m x)\cos(\lambda_m x)dx + \ldots + \int_{0}^{L} C_m\cos(\lambda_n x)\cos(\lambda_m x)dx\right]$$

$$-2\frac{q}{k}\left[\frac{sen\lambda_n x}{\lambda_n}\right]_0^L = 2\left[\frac{x}{2}+\frac{sen(2\lambda x)}{4\lambda}\right]_0^L = 2\left[\frac{L}{2}+\frac{sen(2\lambda L)}{4\lambda}\right]C_n$$

$$-2\frac{q}{k\lambda_n}sen(\lambda_n L) = 2\left[\frac{L}{2}+\frac{sen(2\lambda L)}{4\lambda}\right]C_n$$

$$C_n = \frac{-\,{}^{q}\!/_{k\lambda_n}\,sen(\lambda_n L)}{{}^{L}\!/_{2}+{}^{sen(2\lambda L)}\!/_{4\lambda}} = \frac{-4q\,sen(\lambda_n L)}{[2L\lambda + sen(2\lambda L)]k}$$

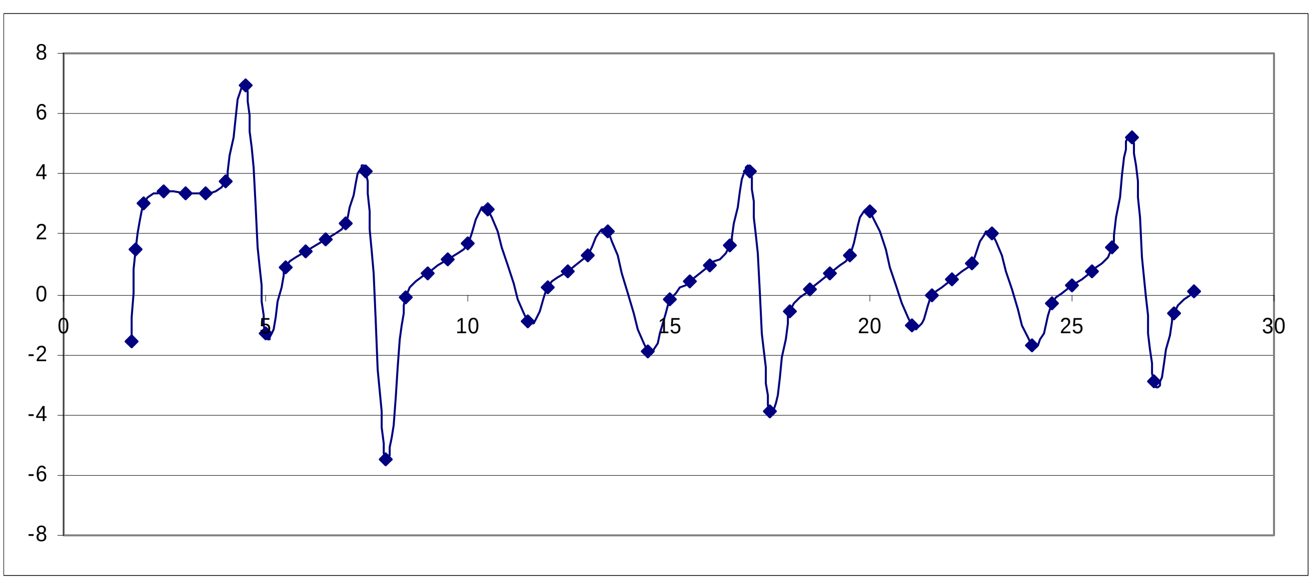
$$A_n = \frac{C_n}{\lambda_n\left[-\,{}^{h}\!/_{k\lambda_n}\cosh(\lambda_n L)+senh(\lambda_n L)\right]}$$

$$\theta_2 = \sum_{n=1}^{7} A_n\left[-\frac{h}{k\lambda}sen(\lambda x)+\cos(\lambda x)\right]\left[-\frac{h}{k\lambda}senh(\lambda y)+\cosh(\lambda y)\right]$$

$$\theta = \sum_{n=1}^{7} A_n\left[-\frac{h}{k\lambda}sen(\lambda x)+\cos(\lambda x)\right]\left[-\frac{h}{k\lambda}senh(\lambda y)+\cosh(\lambda y)\right]$$
$$+\sum_{n=1}^{7} A_n\left[-\frac{h}{k\lambda}sen(\lambda y)+\cos(\lambda y)\right]\left[-\frac{h}{k\lambda}senh(\lambda x)+\cosh(\lambda x)\right]$$

Para el ejercicio se toma un dx=dy=L/10

Se determina el valor de λ graficando la curva y utilizando los promeros 7 valores. Se muestra la grafica a continuación:



VALOR λ	ERROR
3,47185578125	9,18056E-09
10,34669218750	6,7104E-07
17,08149375000	9,74928E-07
23,69009500000	-4,03464E-07
30,20843125000	2,59037E-07
36,66570000000	7,1586E-07
43,08144593750	8,6996E-08

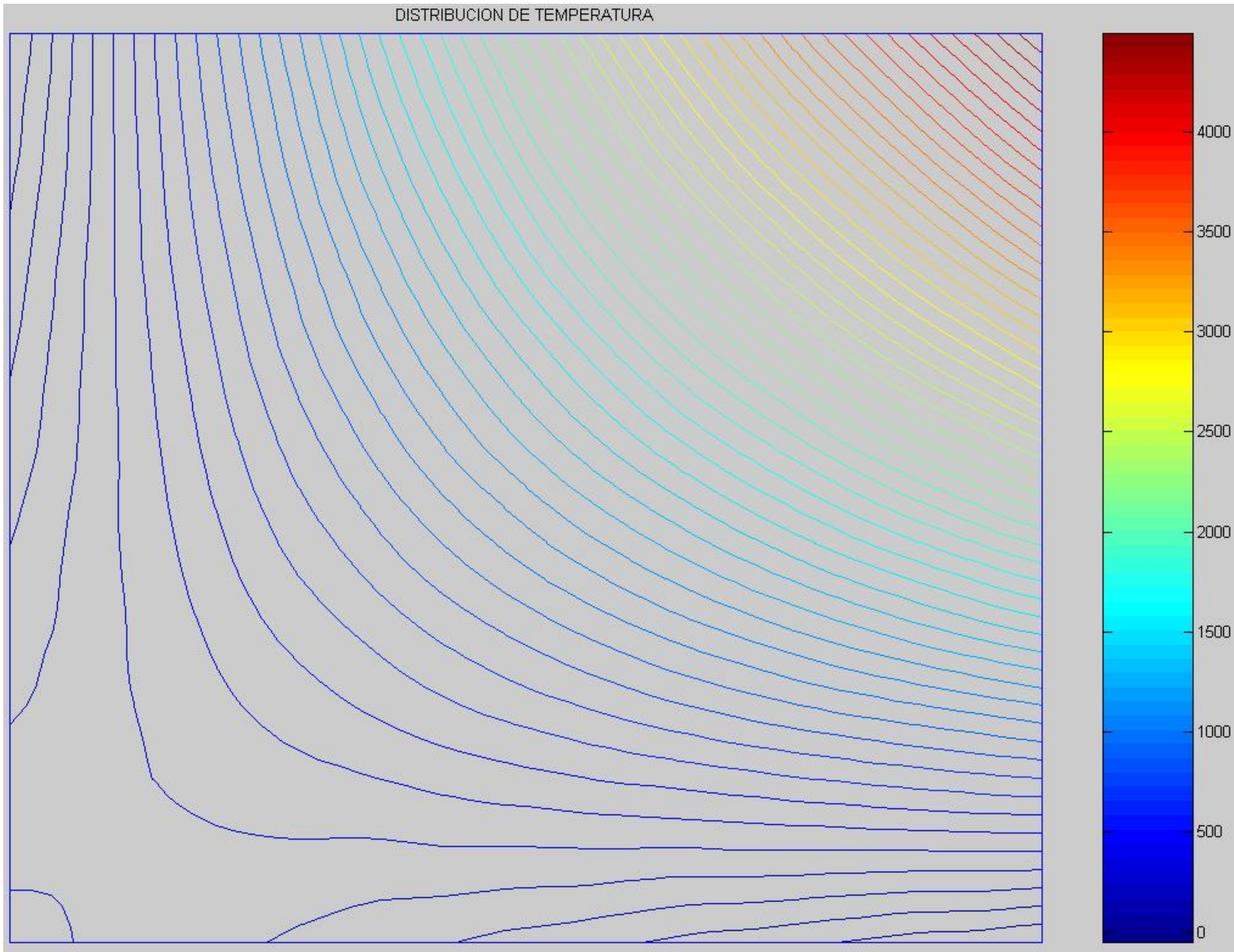
A continuación se muestra el programa en MATLAB del desarrollo de la solucion

```
Clc
clear
q=1200; h=25; k=1.2; Too=300; L=.50; H=L; dx=L/10; dy=dx;
lambda=[3.47185578125 10.3466921875 17.08149375 23.690095 30.20843125 36.6657
43.0814459375];
j=0;
for y=0:dy:H
   for x=0:dx:L
      teta1=0; teta2=0;
      for i=1:2
         Cn=(-4*q*sin(lambda(i)*L))/(k*(2*L*lambda(i)+sin(2*L*lambda(i))));
         An=Cn/(lambda(i)*(-h/(k*lambda(i))*cosh(lambda(i)*L)+sinh(lambda(i)*L)));
         teta1=teta1+An*(-h/(k*lambda(i))*sin(lambda(i)*y)+cos(lambda(i)*y))*(-
h/(k*lambda(i))*sinh(lambda(i)*x)+cosh(lambda(i)*x));
         teta1=teta1+An*(-h/(k*lambda(i))*sin(lambda(i)*x)+cos(lambda(i)*x))*(-
h/(k*lambda(i))*sinh(lambda(i)*y)+cosh(lambda(i)*y));
      end
      teta=teta1+teta2;
      j=j+1;
      T(j)=teta+Too;
   end
end
%
x = [0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;];
%
y = [0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;
   1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;
   2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;
   3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;
   4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;
   5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;
   6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;
   7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;
   8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;
   9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;
   10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;];
```

```
%
% GRAFICA DE LA SOLUCIÓN SOBRE EL MODELO
%
% Genera distribución uniforme de puntos
xi=0:dx/5:L;yi=0:dy/5:H;
[xi,yi]=meshgrid(xi,yi);
% Interpola la solución en los puntos dentro del modelo
FIi=griddata(x,y,T,xi,yi,'cubic');
% Isocurvas a dibujar
mayor=0;menor=1000;
for i=1:121
  a=T(i);
  if a>mayor
     mayor=a;
 elseif a<menor
     menor=a;
   end
end
b=(mayor-menor)/50;
for i=1:50
  ISO_FI(i)=menor+i*b;
end
% Grafica la función
contour(xi,yi,FIi,ISO_FI);colorbar; axis off
title('DISTRIBUCION DE TEMPERATURA');
% Dibujo del modelo
line([0,L,L,0,0],[0,0,H,H,0]);
% Determinacion de Transferencia de Calor
Qc=0;
for i=2:10
   Qc=Qc+h*dx*(T(i)-Too);
end
Qconveccion=2*(h*dx/2*(T(1)-Too)+Qc+h*dx/2*(T(11)-Too))
```

Se obtiene un valor de Qconveccion en suma de las dos caras de : -5.0233e+003W/m

La grafica de Isotermas es la siguiente:


DISTRIBUCION DE TEMPERATURA
4000
3500
3000
2500
2000
1500
1000
500
0

Ejercicio 1. METODO DIFERENCIA FINITA

Modo Interno.

$$T_e - T_o + \left(\frac{\Delta x}{\Delta y}\right)^2 (T_N + T_1) - 2T_p\left[1 + \left(\frac{\Delta x}{\Delta y}\right)^2\right] = 0$$

Borde Convectivo.

$$q_{T_0-T_p} = k\frac{\Delta y}{2}\left(\frac{T_0 - T_p}{\Delta x}\right), \qquad q_{T_e-T_p} = k\frac{\Delta y}{2}\left(\frac{T_e - T_p}{\Delta x}\right),$$

$$q_{T_N-T_p} = k\Delta x\left(\frac{T_N - T_p}{\Delta y}\right), \qquad q_{T_\infty-T_p} = h\Delta x(T_\infty - T_p),$$

$$\dot{Q} = q'' \Delta x$$

$$k\frac{\Delta y}{\Delta x}\left(\frac{T_0 - T_p}{2}\right) + k\frac{\Delta y}{\Delta x}\left(\frac{T_e - T_p}{2}\right) + k\frac{\Delta x}{\Delta y}(T_N - T_p) + h\Delta x(T_\infty - T_p) + q''\Delta x$$

$$\frac{k}{2}\frac{\Delta y}{\Delta x}(T_0 + T_e) + k\frac{\Delta x}{\Delta y}T_N - T_p\left[k\frac{\Delta y}{\Delta x} + k\frac{\Delta x}{\Delta y} + h\Delta x\right] + h\Delta x T_\infty + q'\Delta x = 0$$

$$\frac{2}{k}\frac{\Delta y}{\Delta x} \rightarrow \left(\frac{\Delta y}{\Delta x}\right)^2 (T_0 + T_e) + 2T_N - T_p\left[2\left(\frac{\Delta x}{\Delta x}\right)^2 + 2 + 2\frac{h}{k}\Delta y\right] + 2\frac{h}{k}\Delta y T_\infty + \frac{2q'}{k}\Delta y = 0$$

Para adiabático: $q' = 0$; $h = 0$.

Para convectivo: $q' = 0$.

Borde con Transferencia de Calor.

$$\dot{Q} = q'\frac{\Delta y}{2}; \qquad q_{T_s-T_p} = k\frac{(T_s - T_p)}{\Delta y}\frac{\Delta x}{2}$$

$$q_{T_N-T_p} = k\frac{(T_N - T_p)}{\Delta y}\frac{\Delta x}{2}; \qquad q_{T_0-T_p} = k\Delta y\frac{(T_0 - T_p)}{\Delta x}$$

$$\frac{k}{2}\frac{\Delta x}{\Delta y}(T_s - T_p) + \frac{k}{2}\frac{\Delta x}{\Delta y}(T_N - T_p) + k\frac{\Delta y}{\Delta x}(T_0 - T_p) + q'\Delta y = 0$$

$$\frac{k}{2}\frac{\Delta x}{\Delta y}\left(T_s+T_N\right)+k\frac{\Delta y}{\Delta x}T_0-T_p\left(k\frac{\Delta x}{\Delta y}+k\frac{\Delta y}{\Delta x}\right)+q'\Delta y=0$$

$$\frac{2}{k}\frac{\Delta x}{\Delta y}\rightarrow\left(\frac{\Delta x}{\Delta y}\right)^2\left(T_s+T_N\right)+2T_0-2T_p\left[\left(\frac{\Delta x}{\Delta y}\right)^2+1\right]+\frac{2q'}{k}\left(\Delta x\right)=0$$

Nodo con convección y flujo de calor.

$$\dot{Q}=q'\frac{\Delta x}{2};\qquad q_{T_\infty-T_p}=h\frac{\Delta x}{2}\left(T_\infty-T_p\right);$$

$$q_{T_0-T_p}=k\frac{\Delta y}{2}\left(\frac{T_0-T_p}{\Delta x}\right)\qquad q_{T_N-T_p}=k\frac{\Delta x}{2}\left(\frac{T_N-T_p}{\Delta y}\right)$$

$$\frac{k}{2}\frac{\Delta y}{\Delta x}\left(T_0-T_p\right)+\frac{k}{2}\frac{\Delta x}{\Delta y}\left(T_N-T_p\right)+h\frac{\Delta x}{2}\left(T_\infty-T_p\right)+q'\frac{\Delta x}{2}=0$$

$$\frac{k}{2}\left[\frac{\Delta y}{\Delta x}T_0+\frac{\Delta x}{\Delta y}T_N\right]+h\frac{\Delta x}{2}T_\infty-T_p\left[\frac{k}{2}\left(\frac{\Delta y}{\Delta x}+\frac{\Delta x}{\Delta y}\right)+h\frac{\Delta x}{2}\right]+q'\frac{\Delta x}{2}=0$$

$$\frac{2}{k}\frac{\Delta y}{\Delta x}\rightarrow\left(\frac{\Delta y}{\Delta x}\right)^2T_0+T_N+\frac{h}{k}\Delta yT_\infty-T_p\left[\left(\frac{\Delta y}{\Delta x}\right)^2+1+\frac{h}{k}\Delta y\right]+\frac{q'}{k}\left(\Delta y\right)=0$$

Nodo con borde adiabático.

$$q_{T_e-T_p}=k\Delta y\left(\frac{T_e-T_p}{\Delta x}\right);\qquad q_{T_s-T_p}=k\Delta x\left(\frac{T_s-T_p}{\Delta y}\right)$$

$$k\frac{\Delta y}{\Delta x}\left(T_e-T_p\right)+k\frac{\Delta x}{\Delta y}\left(T_s-T_p\right)=0$$

$$\frac{2}{k}\frac{\Delta y}{\Delta x}\rightarrow 2\left[\left(\frac{\Delta y}{\Delta x}\right)^2T_e+T_s\right]-2T_p\left[\left(\frac{\Delta y}{\Delta x}\right)^2+1\right]=0$$

$$q_{T_s-T_p}=k\frac{\Delta x}{2}\left(\frac{T_s-T_p}{\Delta y}\right);\qquad \dot{Q}=q''\left(\frac{\Delta x}{2}\frac{\Delta y}{2}\right)\frac{1}{2}$$

$$\frac{k}{2}\frac{\Delta x}{\Delta y}\left(T_s-T_p\right)+q''\frac{\Delta x\Delta y}{8}=0$$

$$\frac{2}{k}\frac{\Delta y}{\Delta x}\rightarrow T_s-T_p+\frac{q''}{4k}\left(\Delta y\right)^2=0$$

$$q_{T_e - T_p} = k\frac{\Delta y}{2}\left(\frac{T_e - T_p}{\Delta x}\right); \qquad q_{T_\infty - T_p} = h\frac{\Delta x}{2}\left(T_\infty - T_p\right)$$

$$\frac{k}{2}\frac{\Delta y}{\Delta x}\left(T_e - T_p\right) + \frac{h}{2}\Delta x\left(T_\infty - T_p\right) = 0$$

$$\frac{k}{2}\frac{\Delta y}{\Delta x}T_e + \frac{h}{2}\Delta x T_\infty - T_p\left(\frac{k}{2}\frac{\Delta y}{\Delta x} + h\frac{\Delta x}{2}\right) = 0$$

$$\frac{2}{k}\frac{\Delta y}{\Delta x} \rightarrow \left(\frac{\Delta y}{\Delta x}\right)^2 T_e + \frac{h}{k}\Delta y T_\infty - T_p\left[\left(\frac{\Delta y}{\Delta x}\right)^2 + \frac{h}{k}\Delta y\right] = 0$$

Para el ejercicio se tomo Δy= Δx

Nodo Interior.

$$T_{N,n+1} + T_{S,n-1} + T_{m+1,E} + T_{m-1,O} - 4T_{p:m,n} = 0$$

Nodo como una superficie plana con convección.

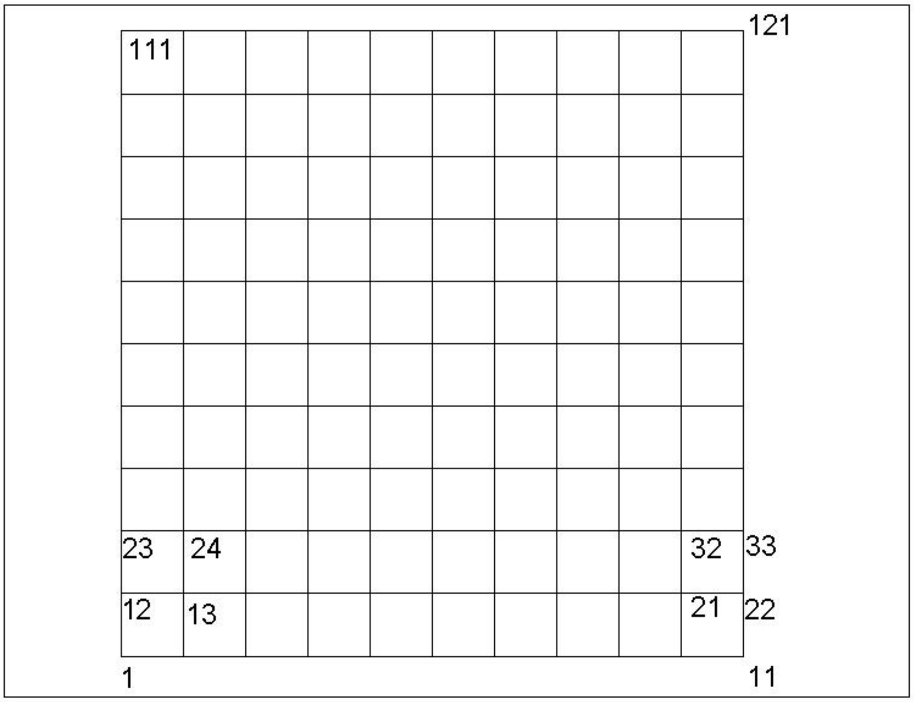
$$\left(2T_{m-1,O} + T_{N,n+1} + T_{S,n-1}\right) + \frac{2h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right)T_{p:m,n} = 0$$

Nodo como una esquina externa con convección.

$$\left(T_{S,n-1} + T_{m-1,O}\right) + 2\frac{h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 1\right)T_{p:m,n} = 0$$

La division se realizo de la siguiente forma:

Dx=dy=L/10



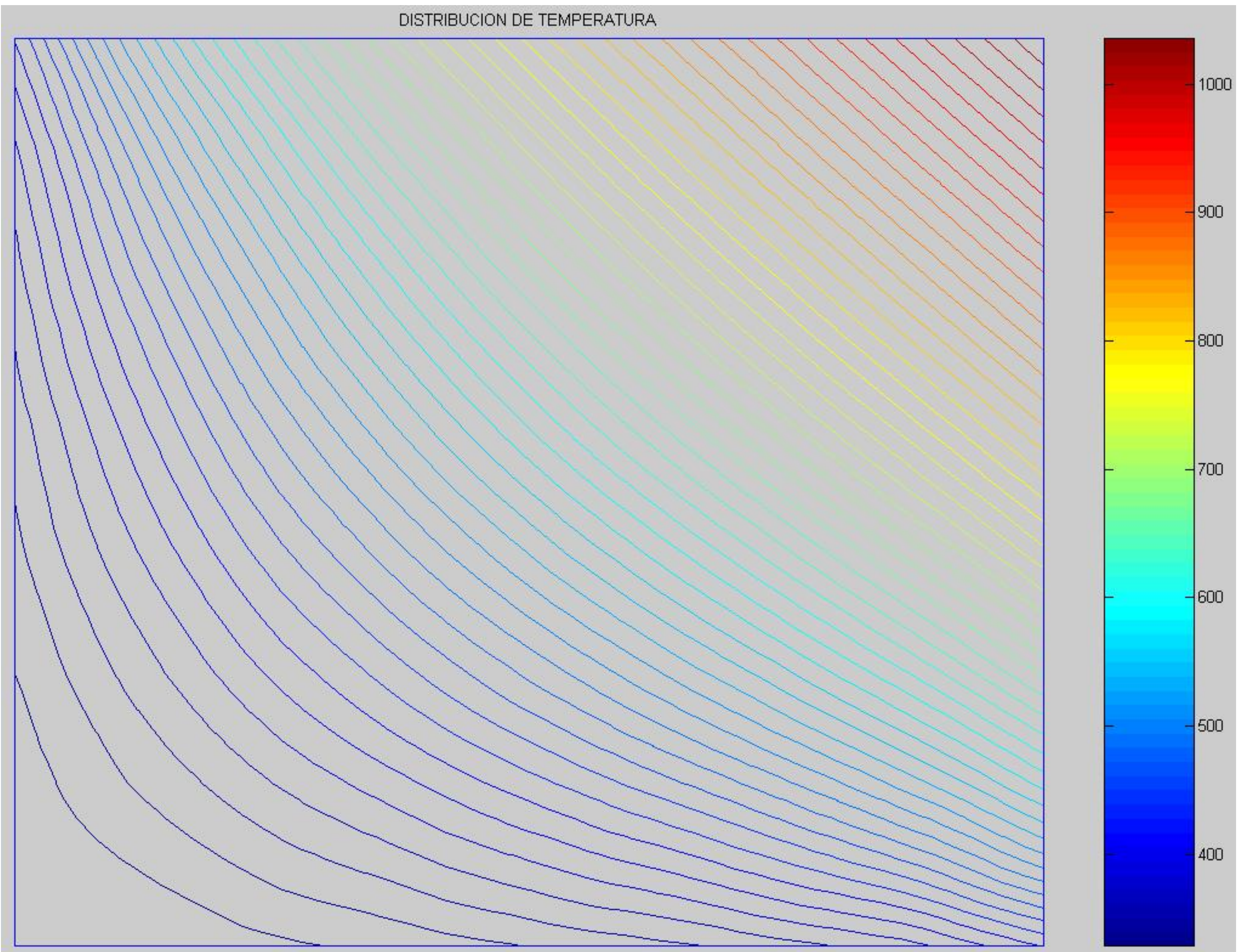
A continuación se muestra el programa de desarrollo del ejercicio:

```
clc
clear
k=1.2; h=25; q=1200; Too=300; L=0.50; H=L; dx=L/10; dy=dx;

%   Coordenadas de los nodos:
%   -------------------------
%
x = [0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;];
%
y = [0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;
   1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;
   2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;
   3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;
   4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;
   5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;
   6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;
   7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;
   8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;
   9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;
   10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;];
```

```
i=1;            %NODO 1 ESQUINA INFERIOR IZQUIERDA
T(i,i)=-2*(h*dx/k+1); T(i,i+1)=1; T(i,i+11)=1; B(i)=-2*h*dx*Too/k;

for i=2:10      %NODOS 2-10 BORDE INFERIOR
   T(i,i-1)=1; T(i,i)=-2*(2+h*dx/k); T(i,i+1)=1; T(i,i+11)=2; B(i)=-2*h*dx*Too/k;
end

i=11;           %NODO 11 ESQUINA INFERIOR DERECHA
T(i,i)=-(h*dx/k+2); T(i,i-1)=1; T(i,i+11)=1; B(i)=-(h*dx*Too/k+q*dx/k);

for i=12:11:100%NODOS 12, 23, 34, 45, 56, 67, 78, 89, 100, BORDE IZQUIERDO
   T(i,i)=-2*(2+h*dx/k); T(i,i+1)=2; T(i,i-11)=1; T(i,i+11)=1; B(i)=-2*h*dx*Too/k;
end

for i=0:11:88  %NODOS 13-21; 24-32; 35-43; 46-54; 57-65; 68-76; 79-87; 90-98; 101-
109; NODOS INTERNOS
   for j=13+i:21+i
      T(j,j-11)=1; T(j,j-1)=1; T(j,j)=-4; T(j,j+1)=1; T(j,j+11)=1; B(j)=0;
   end
end

for i=22:11:110%NODOS 22, 33, 44, 55, 66, 77, 88, 99, 110, BORDE DERECHO
   T(i,i-1)=2; T(i,i)=-4; T(i,i-11)=1; T(i,i+11)=1; B(i)=-2*q*dx/k;
end

i=111;          %NODO 111 ESQUINA SUPERIOR IZQUIERDA
T(i,i)=-(h*dx/k+2); T(i,i+1)=1; T(i,i-11)=1; B(i)=-(h*dx*Too/k+q*dx/k);

for i=112:120  %NODOS 112-120 BORDE SUPERIOR
   T(i,i-1)=1; T(i,i)=-4; T(i,i+1)=1; T(i,i-11)=2; B(i)=-2*q*dx/k;
end

i=121;          %NODO 121 ESQUINA SUPERIOR DERECHA
T(i,i)=-2; T(i,i-1)=1; T(i,i-11)=1; B(i)=-2*q*dx/k;


%Solución del sistema de ecuaciones
FI =inv(T)*(B');

% GRAFICA DE LA SOLUCIÓN SOBRE EL MODELO
%
% Genera distribución uniforme de puntos
xi=0:dx/5:L;yi=0:dy/5:H;
[xi,yi]=meshgrid(xi,yi);
%Interpola la solución en los puntos dentro del modelo
FIi=griddata(x,y,FI,xi,yi,'cubic');
% Isocurvas a dibujar
mayor=0;menor=1000;
for i=1:121
  a=FI(i);
  if a>mayor
     mayor=a;
 elseif a<menor
     menor=a;
   end
end
b=(mayor-menor)/50;
for i=1:50
  ISO_FI(i)=menor+i*b;
```

```
end
% Grafica la función
contour(xi,yi,FIi,ISO_FI);colorbar; axis off
title('DISTRIBUCION DE TEMPERATURA');
% Dibujo del modelo
line([0,L,L,0,0],[0,0,H,H,0]);

% Determinacion de Transferencia de Calor
Qc=0;
for i=2:10
   Qc=Qc+h*dx*(FI(i)-Too);
end
Qconveccion=2*(h*dx/2*(FI(1)-Too)+Qc+h*dx/2*(FI(11)-Too))
```

El valor obtenido por Qconveccion en las dos caras es 1.2000e+003W/m



Ejercicio 2. Solucion Diferencia Finita

Se utilizo para este ejercicio el metodo Implícito de solucion:

Para el borde convectivo:

$$[1+2F_0(2+Bi)]T_p^{\,k+1} - F_0\left(2T_N^{\,k+1} + T_0^{\,k+1} + T_e^{\,k+1}\right) = T_p^{\,k} + 2BiF_0T_\infty$$

Para una esquina de un lado adiabatica y de otro con conveccion:

$$Q_{T_e-T_p} = k\frac{\Delta y}{2}\frac{\left(T_e^{\,k+1} - T_p^{\,k+1}\right)}{\Delta x}; \qquad Q_{T_N-T_p} = k\frac{\Delta x}{2}\frac{\left(T_N^{\,k+1} - T_p^{\,k+1}\right)}{\Delta y};$$

$$\rho C_p\frac{(\Delta x)^2}{4}\frac{\left(T_p^{\,k+1} - T_p^{\,k}\right)}{\Delta_t} = \frac{k}{2}\left(T_e^{\,k+1} - T_p^{\,k+1}\right) + \frac{k}{2}\left(T_N^{\,k+1} - T_p^{\,k+1}\right) + \frac{h}{2}\Delta x\left(T_\infty^{\,k+1} - T_p^{\,k+1}\right)$$

$$T_p^{\,k+1} = \frac{4\Delta t}{\rho C_p(\Delta x)^2}\left[\frac{k}{2}\left(T_e^{\,k+1} - T_p^{\,k+1}\right) + \frac{k}{2}\left(T_N^{\,k+1} - T_p^{\,k+1}\right) + \frac{h}{2}\Delta x\left(T_\infty^{\,k+1} - T_p^{\,k+1}\right)\right] + T_p^{\,k}$$

$$T_p^{\,k+1} = \frac{4\Delta t}{\rho C_p(\Delta x)^2}\left(\frac{k}{2}T_e^{\,k+1} + \frac{k}{2}T_N^{\,k+1} + \frac{h}{2}\Delta x T_\infty^{\,k+1}\right) - T_p^{\,k+1}\left[\frac{4\Delta t}{\rho C_p(\Delta x)^2}\left(\frac{k}{2} + \frac{k}{2} + \frac{h}{2}\Delta x\right)\right] + T_p^{\,k}$$

$$T_p^{\,k+1} = 2F_0\left[T_e^{\,k+1} + T_N^{\,k+1} + BiT_\infty^{\,k+1}\right] - T_p^{\,k+1}F_0(4+2Bi) + T_p^{\,k}$$

$$T_p^{\,k+1}[1+2F_0(2+Bi)] = 2F_0\left[T_e^{\,k+1} + T_N^{\,k+1} + BiT_\infty^{\,k+1}\right] + T_p^{\,k}$$

$$T_p^{\,k+1}[1+2F_0(2+Bi)] - 2F_0\left[T_e^{\,k+1} + T_N^{\,k+1}\right] = T_p^{\,k} + 2F_0BiT_\infty$$

Para un nodo interno:

$$(1+4T_0)T_p^{\,k+1} - F_0\left(T_N^{\,k+1} + T_s^{\,k+1} + T_e^{\,k+1} + T_0^{\,k+1}\right) = T_p^{\,k}$$

Para un nodo de borde, de un lado adiabatico y en la interfase con solidos de distinto k, Cp y uno con generación de calor:

$$Q_{T_e-T_p}=k_1\frac{\Delta y}{2}\frac{T_e-T_p}{\Delta x}+k_2\frac{\Delta y}{2}\frac{T_e-T_p}{\Delta x}=\frac{1}{2}(k_1+k_2)\frac{\Delta y}{\Delta x}(T_e-T_p)$$

$$Q_{T_N-T_p}=k_2\frac{\Delta y}{2}\frac{T_N-T_p}{\Delta y};\qquad Q_{T_s-T_p}=k_1\frac{\Delta x}{2}\frac{T_s-T_p}{\Delta y};$$

$$\dot{Q}_{gen}=q"\frac{\Delta x\Delta y}{4};\qquad \rho C_{p_1}\frac{(\Delta x)^2}{2}+\rho C_{p_2}\frac{(\Delta x)^2}{2}=\rho(\Delta x)^2\frac{1}{2}(C_{p_1}+C_{p_2})$$

$$\rho(\Delta x)^2\frac{1}{2}(C_{p_1}+C_{p_2})=\frac{1}{2}(k_1+k_2)\frac{\Delta y}{\Delta x}(T_e-T_p)+\frac{k_2}{2}\frac{\Delta x}{\Delta y}(T_N-T_p)+\frac{k_1}{2}\frac{\Delta x}{\Delta y}(T_s-T_p)+q"\frac{\Delta x\Delta y}{4}$$

$$T_p^{\,k+1}=\frac{2\Delta t}{\rho(\Delta x)^2(C_{p_1}+C_{p_2})}\left[\frac{1}{2}(k_1+k_2)(T_e-T_p)+\frac{k}{2}(T_N-T_p)+\frac{k_1}{2}(T_s-T_p)+q"\frac{(\Delta x)^2}{4}\right]+T_p^{\,k}$$

$$T_p^{\,k+1}\left[1+F_{0_1}+F_{0_2}\right]-\frac{1}{2}(F_{0_1}+F_{0_2})T_e-\frac{F_{0_2}}{2}T_N-\frac{F_{0_1}}{2}T_p=q"\frac{\Delta t}{4\rho C_p}+T_p^{\,k}$$

Para un nodo interno, en la interfase de solidos de distinto k, Cp y uno con generación de calor:

$$Q_{T_N-T_p}=k_2\frac{\Delta x}{\Delta y}(T_N-T_p);\qquad Q_{T_s-T_p}=k_1\frac{\Delta x}{\Delta y}(T_s-T_p);$$

$$Q_{T_e-T_p}=\frac{1}{2}(k_1+k_2)\frac{\Delta y}{\Delta x}(T_e-T_p);\qquad Q_{T_0-T_p}=\frac{1}{2}(k_1+k_2)\frac{\Delta y}{\Delta x}(T_0-T_p);$$

$$Q_{gen}=q"\frac{\Delta x\Delta y}{2}$$

$$\rho C_p(\Delta x)^2\frac{(T_p^{\,k+1}-T_p^{\,k})}{\Delta t}=k_2(T_N-T_p)+k_1(T_s-T_p)+\frac{1}{2}(k_1+k_2)(T_e-T_p)+\frac{1}{2}(k_1+k_2)(T_0-T_p)+q"\frac{(\Delta x)^2}{2}$$

$$T_p^{\,k+1}=\frac{\Delta t}{\rho C_p(\Delta x)^2}\left[k_2(T_N-T_p)+k_1(T_s-T_p)+\frac{1}{2}(k_1+k_2)(T_e-T_p)+\frac{1}{2}(k_1+k_2)(T_0-T_p)+q"\frac{(\Delta x)^2}{2}\right]+T_p^{\,k}$$

$$T_p^{\,k+1}=\frac{\Delta t}{\rho C_p(\Delta x)^2}\left[k_2T_N+k_1T_s+\frac{1}{2}(k_1+k_2)T_e+\frac{1}{2}(k_1+k_2)T_0\right]+q"\frac{\Delta t}{2\rho C_p}-T_p^{\,k+1}\frac{\Delta t}{\rho C_p(\Delta x)^2}\left[k_2+k_1+\frac{1}{2}(k_1+k_2)+\frac{1}{2}(k_1+k_2)\right]+T_p^{\,k}$$

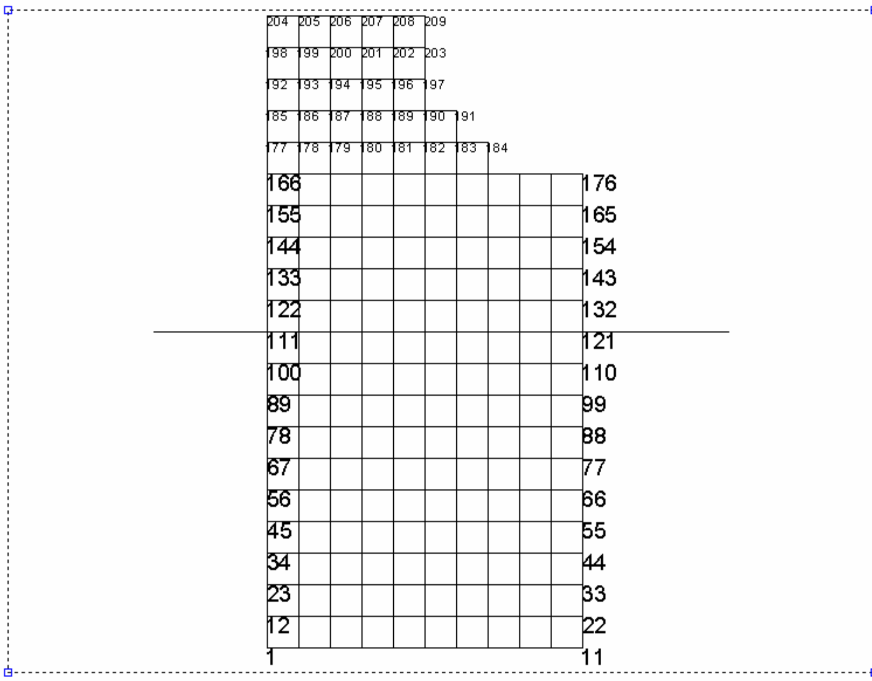
$$T_p^{k+1} = F_{0_2}T_N + F_{0_1}T_s + \frac{1}{2}\left(F_{0_2} + F_{0_1}\right)T_e + \frac{1}{2}\left(F_{0_2} + F_{0_1}\right)T_0 + q'' \frac{\Delta t}{2\rho C_p} - T_p^{k+1}\left[F_{0_2} + F_{0_1} + \frac{1}{2}\left(F_{0_2} + F_{0_1}\right) + \frac{1}{2}\left(F_{0_2} + F_{0_1}\right)\right] + T_p^{k}$$

$$T_p^{k+1}\left[1 + 2\left(F_{0_2} + F_{0_1}\right)\right] - \left[F_{0_2}T_N + F_{0_1}T_s + \frac{1}{2}\left(F_{0_2} + F_{0_1}\right)T_e + \frac{1}{2}\left(F_{0_2} + F_{0_1}\right)T_0\right] = q'' \frac{\Delta t}{2\rho C_p} + T_p^{k}$$

Para un nodo de esquina con conveccion por dos lados:

$$(1 + 4Fo(1 + Bi)).T_p^{K+1} - 2Fo(T_o + T_S) = T_p^{K} + 4BiFoT_{oo}$$

Para resolver el ejercicio se tomo una zona que fuera simetrica con todo el ejercicio y se dividio de la siguiente forma. Hay q notar q existe una leve desviación en la zona radial, Se consideraron los puntos que conformaban ese radio, los demas que quedaban fuera no se consideraron, tomando ese sector con conveccion. La línea media ubicada entre 111-121 muestra la interfase entre los dos solidos.



El programa se desarrollo de la siguiente forma:

```
clc
clear
q=1500; hi=1000; he=25; k1=.5; k2=25; Too=300; dx=.02; dy=dx; dt=300; ro=3000;
cp1=700; cp2=500;
alfa1=k1/(ro*cp1); alfa2=k2/(ro*cp2); Fo1=alfa1*dt/dx^2; Fo2=alfa2*dt/dx^2;
Bie1=he*dx/k1; Bii2=hi*dx/k2; Ti=723;
alfa1L=k1/(ro*(cp1+cp2)/2); alfa2L=k2/(ro*(cp1+cp2)/2); Fo1L=alfa1L*dt/(dx)^2;
Fo2L=alfa2L*dt/(dx)^2;

x = [0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;8*dx;9*dx;10*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;7*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;6*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;
   0*dx;1*dx;2*dx;3*dx;4*dx;5*dx;];
%
y = [0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;0*dy;
   1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;1*dy;
   2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;2*dy;
   3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;3*dy;
   4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;4*dy;
   5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;5*dy;
   6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;6*dy;
   7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;7*dy;
   8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;8*dy;
   9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;9*dy;
   10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;10*dy;
   11*dy;11*dy;11*dy;11*dy;11*dy;11*dy;11*dy;11*dy;11*dy;11*dy;11*dy;
   12*dy;12*dy;12*dy;12*dy;12*dy;12*dy;12*dy;12*dy;12*dy;12*dy;12*dy;
   13*dy;13*dy;13*dy;13*dy;13*dy;13*dy;13*dy;13*dy;13*dy;13*dy;13*dy;
   14*dy;14*dy;14*dy;14*dy;14*dy;14*dy;14*dy;14*dy;14*dy;14*dy;14*dy;
   15*dy;15*dy;15*dy;15*dy;15*dy;15*dy;15*dy;15*dy;15*dy;15*dy;15*dy;
   16*dy;16*dy;16*dy;16*dy;16*dy;16*dy;16*dy;16*dy;
   17*dy;17*dy;17*dy;17*dy;17*dy;17*dy;17*dy;
   18*dy;18*dy;18*dy;18*dy;18*dy;18*dy;
   19*dy;19*dy;19*dy;19*dy;19*dy;19*dy;
   20*dy;20*dy;20*dy;20*dy;20*dy;20*dy;];

for i=1:209
   T(i,i)=Ti;
end
```

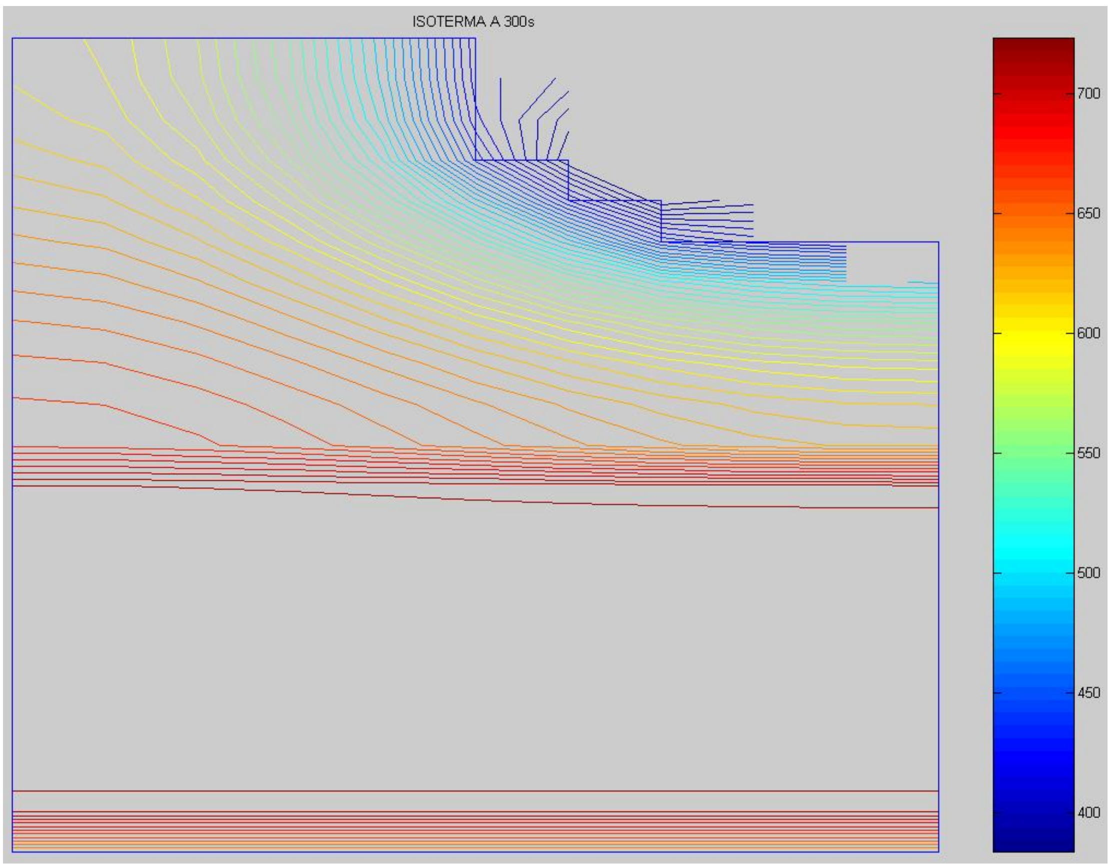
```
%for w=2:6

   i=1;           %NODO 1 CONVECCION Y ADIABATICO
   B(i)=T(i,i)+2*Fo1*Bie1*Too; T(i,i)=1+2*Fo1*(2+Bie1); T(i,i+1)=-2*Fo1; T(i,i+11)=-
2*Fo1;

   for i=2:10     %NODOS 2-10 SOLO CONVECCION
      B(i)=T(i,i)+2*Fo1*Bie1*Too; T(i,i)=1+2*Fo1*(2+Bie1); T(i,i-1)=-Fo1; T(i,i+1)=-
Fo1; T(i,i+11)=-2*Fo1;
   end

   i=11;          %NODO 11 CONVECCION Y ADIABATICO
   B(i)=T(i,i)+2*Fo1*Bie1*Too; T(i,i)=1+2*Fo1*(2+Bie1); T(i,i-1)=-2*Fo1; T(i,i+11)=-
2*Fo1;

   for i=12:11:100%NODOS 12, 23, 34, 45, 56, 67, 78, 89, 100 ADIABATICO
      B(i)=T(i,i); T(i,i)=1+4*Fo1; T(i,i-11)=-Fo1; T(i,i+1)=-2*Fo1; T(i,i+11)=-Fo1;
   end

   for i=0:8      %NODOS   13-21; 24-32; 35-43; 46-54; 57-65; 68-76; 79-87; 90-98;
101-109; CONDUCCION
      for j=13+11*i:21+11*i
         B(j)=T(j,j); T(j,j)=1+4*Fo1; T(j,j-11)=-Fo1; T(j,j-1)=-Fo1; T(j,j+1)=-Fo1;
T(j,j+11)=-Fo1;
      end
   end

   for i=22:11:110%NODOS 22, 33, 44, 55, 66, 77, 88, 99, 110, ADIABATICO
      B(i)=T(i,i); T(i,i)=1+4*Fo1; T(i,i-11)=-Fo1; T(i,i-1)=-2*Fo1; T(i,i+11)=-Fo1;
   end

   i=111;         %NODO 111 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*(cp1+cp2)); T(i,i)=1+2*(Fo1L+Fo2L); T(i,i-11)=-Fo1L;
T(i,i+1)=-(Fo1L+Fo2L); T(i,i+11)=-Fo2L;

   for i=112:120  %NODOS 112-120 CONDUCCION
      B(i)=T(i,i)+q*dt/(ro*(cp1+cp2)); T(i,i)=1+2*(Fo1L+Fo2L); T(i,i-11)=-Fo1L; T(i,i-
1)=-(Fo1L+Fo2L)/2; T(i,i+1)=-(Fo1L+Fo2L)/2; T(i,i+11)=-Fo2L;
   end

   i=121;         %NODO 111 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*(cp1+cp2)); T(i,i)=1+2*(Fo1L+Fo2L); T(i,i-11)=-Fo1L; T(i,i-
1)=-(Fo1L+Fo2L); T(i,i+11)=-Fo2L;

   for i=122:11:166%NODOS 122, 133, 144, 155, 166, ADIABATICO
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-11)=-Fo2; T(i,i+1)=-2*Fo2;
T(i,i+11)=-Fo2;
   end

   for i=0:3      %NODOS   123-131; 134-142; 145-153; 156-164 CONDUCCION
      for j=123+11*i:131+11*i
         B(j)=T(j,j)+q*dt/(ro*cp2); T(j,j)=1+4*Fo2; T(j,j-11)=-Fo2; T(j,j-1)=-Fo2;
T(j,j+1)=-Fo2; T(j,j+11)=-Fo2;
      end
   end

   for i=132:11:165%NODOS 132, 143, 154, 165, ADIABATICO
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-11)=-Fo2; T(i,i-1)=-2*Fo2;
```

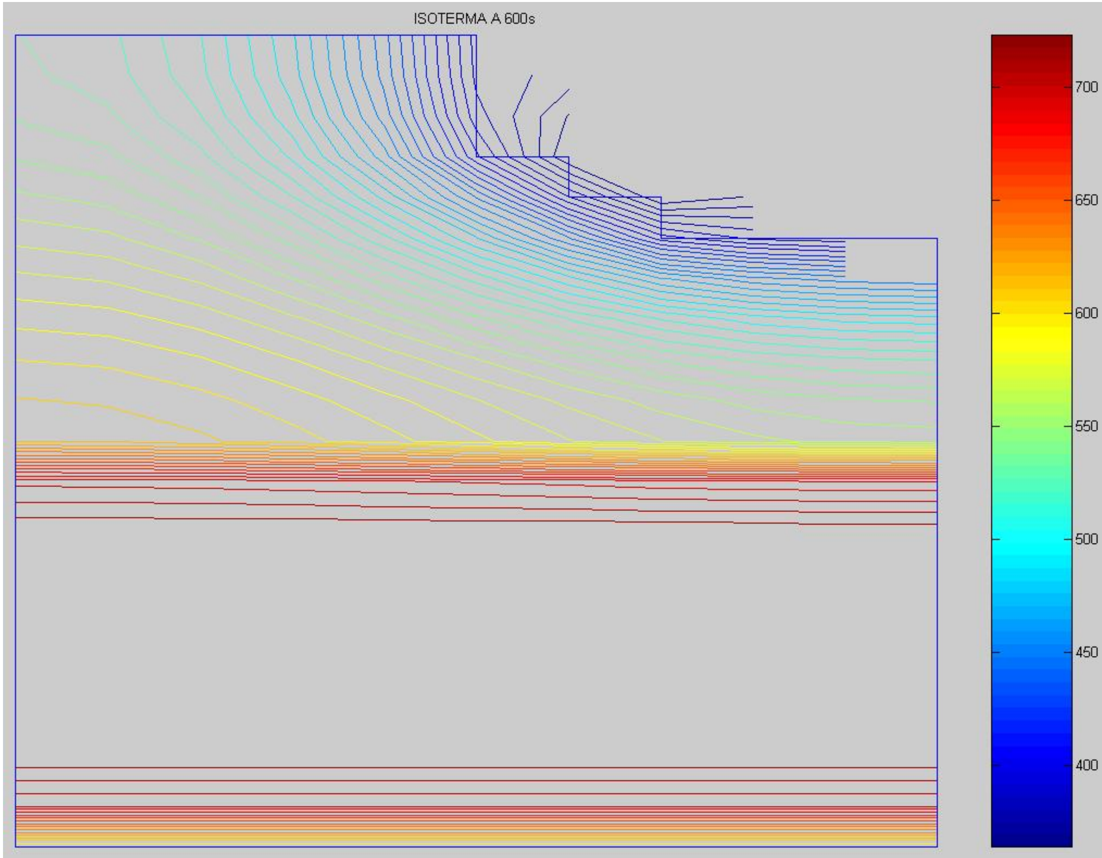
```
T(i,i+11)=-Fo2;
   end

   for i=167:172  %NODOS 167-172 CONDUCCION
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-11)=-Fo2; T(i,i-1)=-Fo2;
T(i,i+1)=-Fo2; T(i,i+11)=-Fo2;
   end

   i=173;
   B(i)=T(i,i)+4*Bii2*Fo2*Too/3+q*dt/(ro*cp2); T(i,i)=1+4*Fo2*(1+Bii2/3); T(i,i-11)=-
4*Fo2/3; T(i,i-1)=-4*Fo2/3; T(i,i+1)=-2*Fo2/3; T(i,i+11)=-2*Fo2/3;

   for i=174:175  %NODOS 174-175 SOLO CONVECCION
      B(i)=T(i,i)+2*Fo2*Bii2*Too+q*dt/(ro*cp2); T(i,i)=1+2*Fo2*(2+Bii2); T(i,i-1)=-
Fo2; T(i,i+1)=-Fo2; T(i,i-11)=-2*Fo2;
   end

   i=176;         %NODOS 176  CONVECCION Y ADIABATICO
   B(i)=T(i,i)+2*Fo2*Bii2*Too+q*dt/(ro*cp2); T(i,i)=1+2*Fo2*(2+Bii2); T(i,i-1)=-2*Fo2;
T(i,i-11)=-2*Fo2;

   i=177;            %NODO 177 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-11)=-Fo2; T(i,i+1)=-2*Fo2;
T(i,i+8)=-Fo2;

   for i=178:182  %NODOS 178-182 CONDUCCION
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-11)=-Fo2; T(i,i-1)=-Fo2;
T(i,i+1)=-Fo2; T(i,i+8)=-Fo2;
   end

   i=183;         %NODOS 183  CONVECCION Y ADIABATICO
   B(i)=T(i,i)+4*Bii2*Fo2*Too/3+q*dt/(ro*cp2); T(i,i)=1+4*Fo2*(1+Bii2/3); T(i,i-11)=-
4*Fo2/3; T(i,i-1)=-4*Fo2/3; T(i,i+1)=-2*Fo2/3; T(i,i+8)=-2*Fo2/3;

   i=184;      %NODOS 184  CONVECCION Y CONVECCION
   B(i)=T(i,i)+4*Bii2*Fo2*Too+q*dt/(ro*cp2); T(i,i)=1+4*Fo2*(1+Bii2); T(i,i-11)=-
2*Fo2; T(i,i-1)=-2*Fo2;

   i=185;            %NODO 185 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-8)=-Fo2; T(i,i+1)=-2*Fo2;
T(i,i+7)=-Fo2;

   for i=186:189  %NODOS 186-189 CONDUCCION
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-8)=-Fo2; T(i,i-1)=-Fo2;
T(i,i+1)=-Fo2; T(i,i+7)=-Fo2;
   end

   i=190;         %NODOS 190  CONVECCION Y CONVECCION
   B(i)=T(i,i)+4*Bii2*Fo2*Too/3+q*dt/(ro*cp2); T(i,i)=1+4*Fo2*(1+Bii2/3); T(i,i-8)=-
4*Fo2/3; T(i,i-1)=-4*Fo2/3; T(i,i+1)=-2*Fo2/3; T(i,i+7)=-2*Fo2/3;

   i=191;         %NODOS 191  CONVECCION Y CONVECCION
   B(i)=T(i,i)+4*Bii2*Fo2*Too+q*dt/(ro*cp2); T(i,i)=1+4*Fo2*(1+Bii2); T(i,i-8)=-2*Fo2;
T(i,i-1)=-2*Fo2;

   i=192;            %NODO 192 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-7)=-Fo2; T(i,i+1)=-2*Fo2;
T(i,i+6)=-Fo2;
```

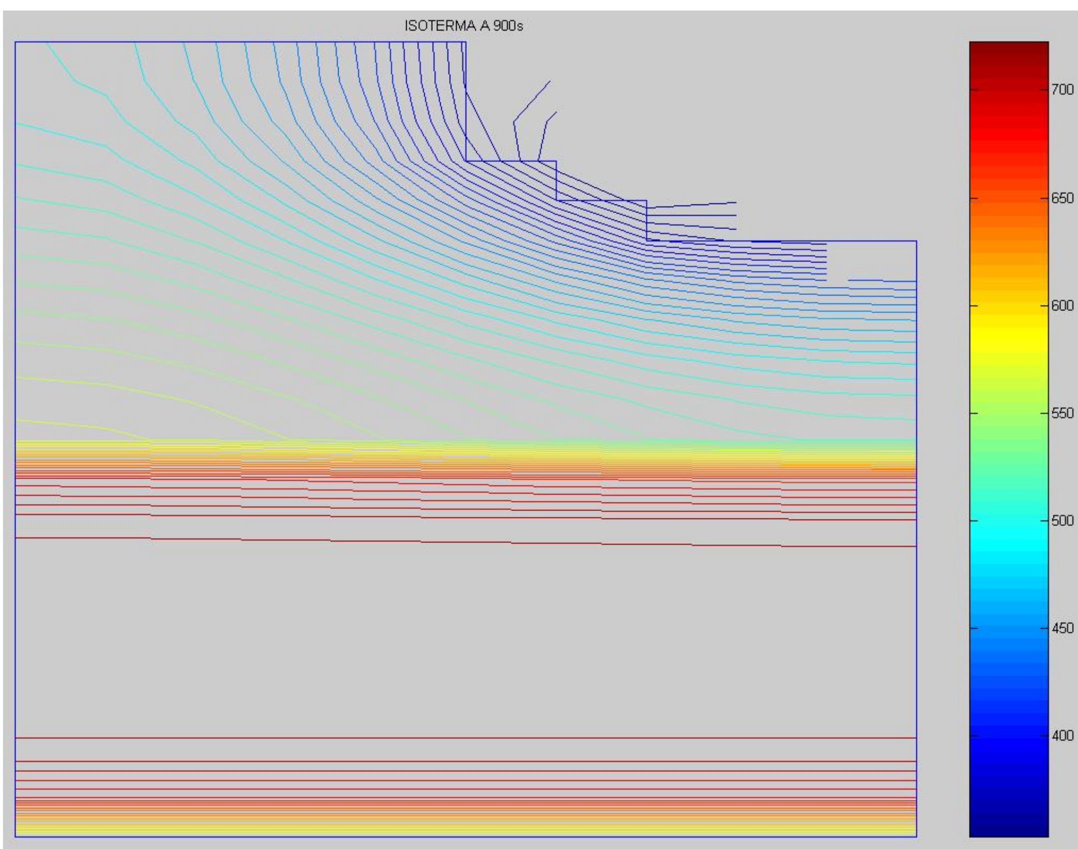
```
   for i=193:196  %NODOS 193-196 CONDUCCION
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-7)=-Fo2; T(i,i-1)=-Fo2;
T(i,i+1)=-Fo2; T(i,i+6)=-Fo2;
   end

   i=197;             %NODOS 197 CONVECCION
   B(i)=T(i,i)+2*Fo2*Bii2*Too+q*dt/(ro*cp2); T(i,i)=1+2*Fo2*(2+Bii2); T(i,i-1)=-2*Fo2;
T(i,i-7)=-Fo2; T(i,i+6)=-Fo2;

   i=198;             %NODO 198 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-6)=-Fo2; T(i,i+1)=-2*Fo2;
T(i,i+6)=-Fo2;

   for i=199:202  %NODOS 199-202 CONDUCCION
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-6)=-Fo2; T(i,i-1)=-Fo2;
T(i,i+1)=-Fo2; T(i,i+6)=-Fo2;
   end

   i=203;             %NODOS 203 CONVECCION
   B(i)=T(i,i)+2*Fo2*Bii2*Too+q*dt/(ro*cp2); T(i,i)=1+2*Fo2*(2+Bii2); T(i,i-1)=-2*Fo2;
T(i,i-6)=-Fo2; T(i,i+6)=-Fo2;

   i=204;             %NODO 204 ADIABATICO
   B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i+1)=-2*Fo2; T(i,i-6)=-2*Fo2;

   for i=205:208  %NODOS 205-208 ADIABATICO
      B(i)=T(i,i)+q*dt/(ro*cp2); T(i,i)=1+4*Fo2; T(i,i-1)=-Fo2; T(i,i+1)=-Fo2; T(i,i-
6)=-2*Fo2;
   end

   i=209;          %NODOS 209  CONVECCION Y ADIABATICO
   B(i)=T(i,i)+2*Fo2*Bii2*Too; T(i,i)=1+2*Fo2*(2+Bii2); T(i,i-1)=-2*Fo2; T(i,i-6)=-
2*Fo2;

   %end

%Solución del sistema de ecuaciones
FI =inv(T)*(B');
%
% GRAFICA DE LA SOLUCIÓN SOBRE EL MODELO
%
% Genera distribución uniforme de puntos
xi=0:dx:.20;yi=0:dx:.40;
[xi,yi]=meshgrid(xi,yi);
% Interpola la solución en los puntos dentro del modelo
FIi=griddata(x,y,FI,xi,yi,'cubic');
% Isocurvas a dibujar
mayor=0;menor=723;
for i=1:209
  a=FI(i);
  if a>mayor
     mayor=a;
 elseif a<menor
     menor=a;
   end
end
b=(mayor-menor)/50;
for i=1:50
  ISO_FI(i)=menor+i*b;
```

```
end
% Grafica la función
contour(xi,yi,FIi,ISO_FI);colorbar; axis off
title('ISOTERMA A 300s');
% Dibujo del modelo
line([0,.20,.20,.14,.14,.12,.12,.10,.10,0,0],[0,0,.30,.30,.32,.32,.34,.34,.40,.40,0]);
```

Para obtener las isortermas se variaba el dt en un intervalo de 300s (5min) hasta 1800s (30min) y se volvia a correr el programa con el nuevo valor, obteniendose asi distintas graficas, es interesante resaltar el cambio de temperatura en la interfase. Entre dos solidos con un valor de k muy diferentes.




ISOTERMA A 600s
700
650
600
550
500
450
400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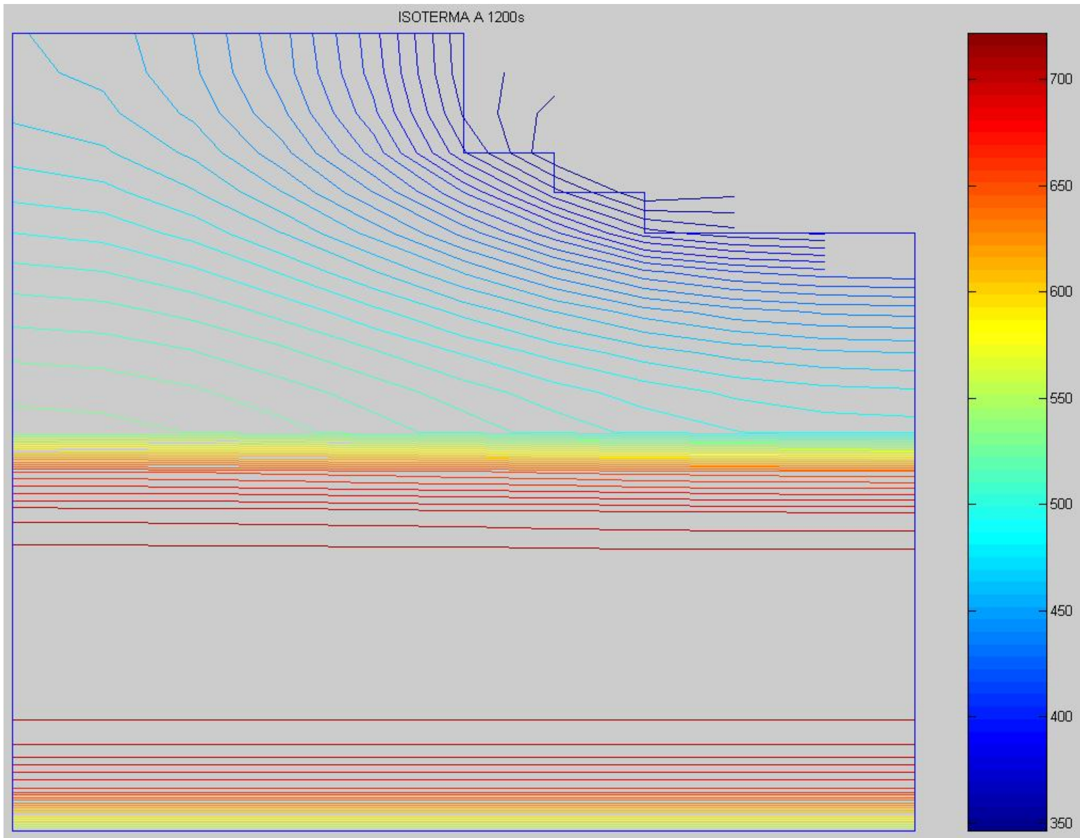
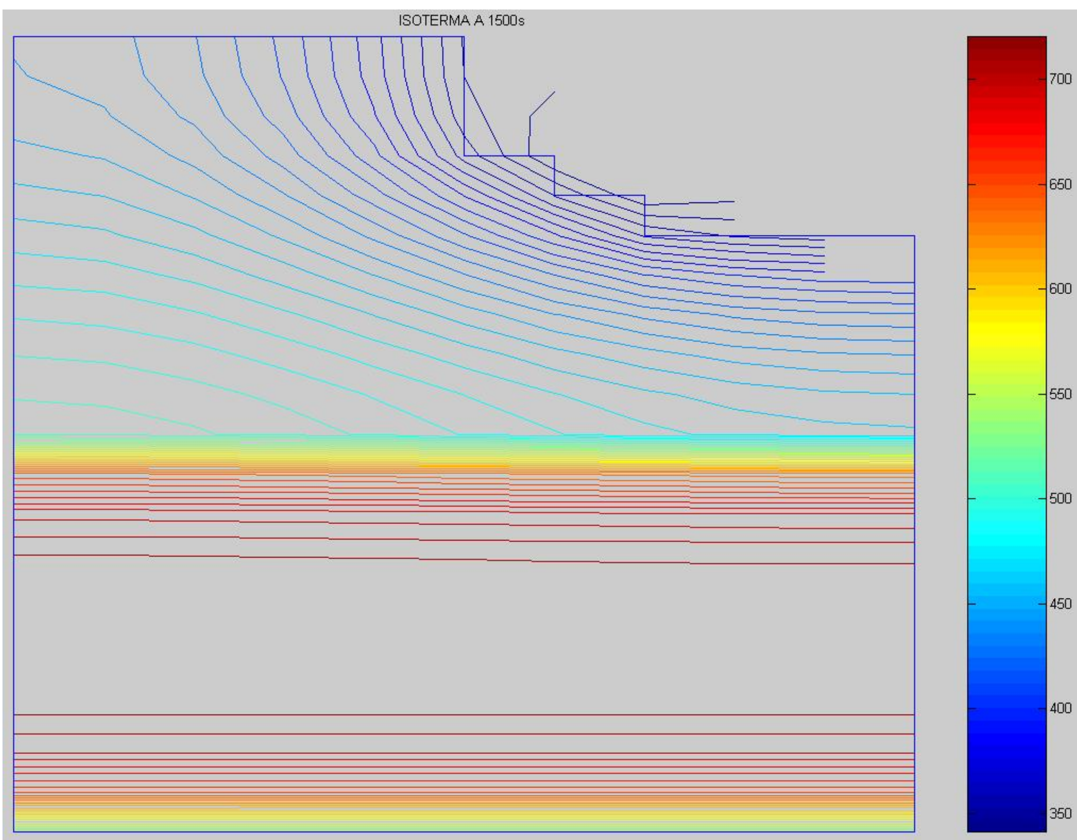
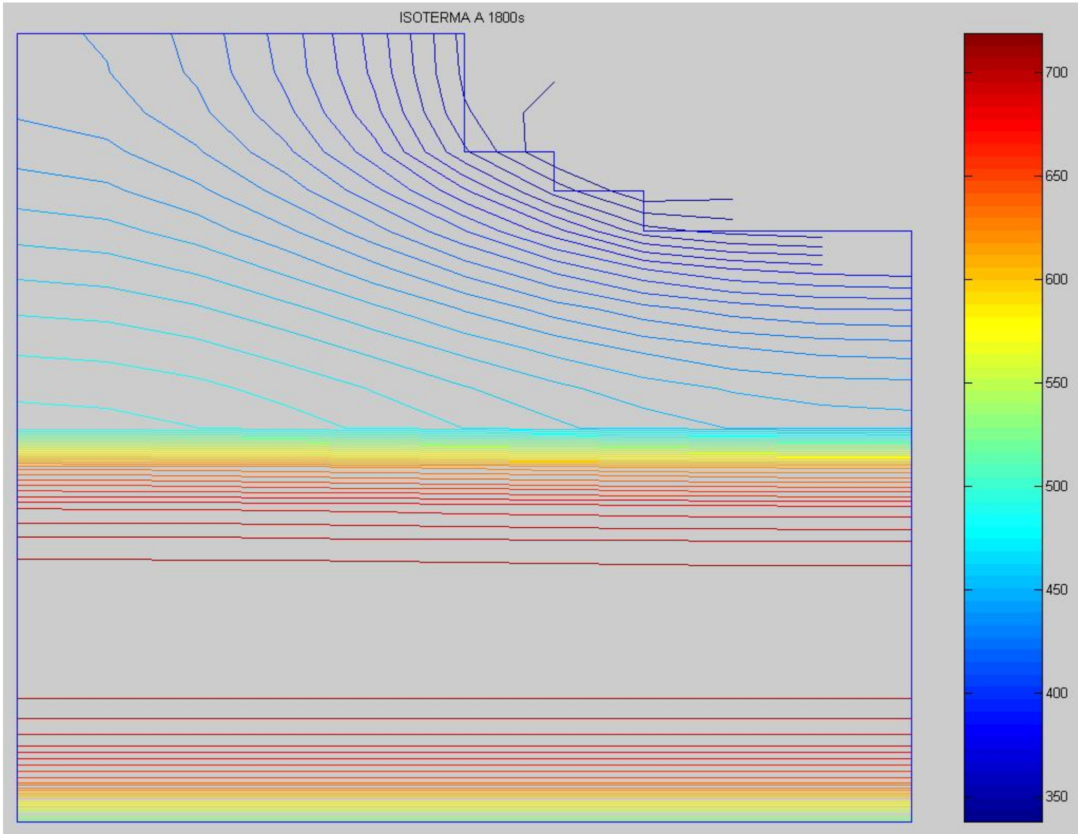
650
600
550
500
450
400


ISOTERMA A 1200s
700
650
600
550
500
450
400
350


ISOTERMA A 1500s
700
650
600
550
500
450
400
350


ISOTERMA A 1800s
700
650
600
550
500
450
400
350